§12.6 Quadratic Surfaces

Ex: Understand the Surface w/ equation

$$x^2+y^2-2x-6y-z+10=0$$

Sol: First we will rewrite the equation: $(x^2-2x)+(y^2-6y)-z+10=0$
(complete the square)

iff $(x^2+2(-1)x+(-1)^2-(-1)^2)+(y^2+2(-3)y+(-3)^2-(-3)^2) - z+10=0$

* $(a+b)^2$ iff $(x-1)^2-(-1)^2+(y-3)^2-(-3)^2-z+10$

$=a^2+2ab+b^2$ iff $(x-1)^2+(y-3)^2-z=0$

now we analyze the equation via Cross-section

when $z=k$ (constant): $(x-1)^2+(y-3)^2-k=0$

$(x-1)^2+(y-3)^2=k$ ← This is an ellipse (or a point or empty) at $k=0$

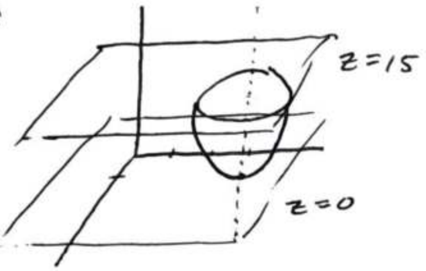
when $y=k$: $(x-1)^2+(k-3)^2-z=0$

$z=(x-1)^2+(k-3)^2$ ← parabola! (Upward Facing)

when $x=k$: $(k-1)^2+(y-3)^2-z=0$

$z=(y-3)^2+(k-1)^2$ ← parabola (Upward facing)

Picture:



Equation	Name
$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z}{c}=0$	Elliptic Paraboloid
$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$	Ellipsoid
$\frac{x^2}{a^2}-\frac{y^2}{b^2}-\frac{z}{c}=0$	Hyperbolic Paraboloid
$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$	One-Sheet Hyperboloid
$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=0$	Cone
$-\frac{x^2}{a^2}-\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$	Two-Sheet Hyperboloid

Conic Sections

Ellipses: $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

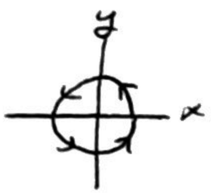
Parabolas: $\frac{x^2}{a^2}+\frac{y}{c}=0$

Hyperbolas: $\frac{x^2}{a^2}-\frac{y^2}{b^2}=0$

§ 13.1 Space Curves

A space curve is a function $\vec{r}: I \to \mathbb{R}^n$

Ex: The Helix is the curve $\vec{r}(t) = \langle \cos(t), \sin(t), t \rangle$



Definition: The limit of space curve $\vec{r}(t) = \langle x(t), y(t), z(t) \rangle$ at time $t=a$ is the component limit provided each component limits as $t \to a$

i.e. $\lim\limits_{t\to a} \vec{r}(t) = \lim\limits_{t\to a} \langle x(t)\ \ y(t)\ \ z(t) \rangle$

$= \langle \lim\limits_{t\to a} x(t),\ \lim\limits_{t\to a} y(t)\ \ \lim\limits_{t\to a} (z(t)) \rangle$

Exercise: Compute $\lim\limits_{t\to \frac{7\pi}{16}} \vec{r}(t)$ for $\vec{r}(t) = \langle (1+5\sin(20t)) \cos(8t), (1+5\sin(20t)) \sin(8t), \cos(20t) \rangle$